Online-appendix to "Brain Drain, Fiscal Competition, and Public Education Expenditure"

Proof of Lemma 1

According to (1), the demand for the two factors S^j and L^j at location $j = H, F$ is given by

$$w_S^j = A^j \beta \left(L^j / S^j\right)^{1-\beta}, \quad w_L^j = A^j (1-\beta) \left(L^j / S^j\right)^{-\beta}, \tag{A1}$$

which implies that

$$\omega^j = \frac{\beta}{1-\beta} \frac{L^j}{S^j}. \tag{A2}$$

Combining (4) and (A2), we obtain

$$\frac{S^j}{L^j} = \frac{\beta G^j}{\phi (1-\beta)} \tag{A3}$$

for the skill intensity in equilibrium.

In order to characterize equilibrium employment, we first consider a scenario without migration. With $\mu^j = \mu^k = 0$, we have $S^j = (1-\bar{e})(1-L^j)G^j$ and thus, according to (A3), in equilibrium,

$$L_0^j = 1 - \beta, \quad S_0^j = \beta G^j / \phi. \tag{A4}$$

(Non-migration equilibrium values are indicated by subscript 0.) If migration is allowed for, the equilibrium levels of employment are:

$$L^j = L_0^j \left[1 - \mu^j + \mu^k G^k / G^j\right], \quad S^j = S_0^j \left[1 - \mu^j + \mu^k G^k / G^j\right]. \tag{A5}$$

To see this, substitute (2) for S^j into (A3) and solve for L^j. Then substitute L^j into (A3) and solve for S^j.

Substituting (A3) into (A1), we further obtain

$$w_S^j = b A^j \left(\phi / G^j\right)^{1-\beta}, \tag{A6}$$

with $b = \beta^\beta (1-\beta)^{1-\beta}$. Education expenditure is financed by a wage income tax with tax rate τ^j. The budget constraint in country j is thus:

$$G^j = \tau^j Y^j = \tau^j \left[w_S^j S^j + w_L^j L^j \right]. \tag{A7}$$

The tax burden per efficiency unit of high-skilled labor $\tau^j w_S^j$ is equal to the ratio $w_S^j G^j / Y^j$. This implies for the closed economy that $\tau^j w_S^j = w_S^j G^j / Y_0^j = \phi$. (Use $L_0^j = 1-\beta$, according to (A4), and note that $1-\beta$ is the income share of low-skilled labor: $1-\beta = w_L^j L_0^j / Y_0^j$. Then $Y_0^j = w_L^j$ and $w_S^j G^j / Y_0^j = \phi$ follows from (4).) Using (A5) in (1), we have $Y^j = Y_0^j \left[1 - \mu^j + \mu^k G^k / G^j\right]$. Hence, under migration,

$$\tau^j w_S^j = \frac{\phi}{1 - \mu^j + \mu^k G^k / G^j}. \tag{A8}$$

A positive net wage requires $w_S^j > \tau_S^j w_S^j$. A sufficient condition is $w_S^j > \phi/(1-\mu^j)$, which, after substitution of (A6) implies (6). Equ. (7) follows from combining (A6) and (A8). Differentiating equ. (7) with respect to μ^H, G^H and G^F proves the final statement in Lemma 1. *QED.*

Proof of Lemma 2

Equ. (11) follows from (4), (A6) and (A8). Partial differentiation of (11) with respect to G^j gives

$$\frac{\partial W^j}{\partial G^j} = \frac{1}{G^j} \left[\beta b A^j \left(G^j/\phi\right)^\beta - \frac{G^j}{[1 - \mu^j + \mu^k G^k / G^j]} - \frac{\mu^k G^k}{[1 - \mu^j + \mu^k G^k / G^j]^2} \right], \tag{A9}$$

and $\partial W^j / \partial G^j = 0$ implicitly determines a unique $G^j = \tilde{G}^j \left(\mu^j, \mu^k; G^k\right)$. Moreover, $\partial^2 W^j / (\partial G^j)^2 |_{G^j = \tilde{G}^j(\cdot)} < 0$. Thus, $G^j = \tilde{G}^j \left(\mu^j, \mu^k; G^k\right)$ is the best response of j to G^k for given μ^j, μ^k.

Applying the implicit function theorem to $\partial W^j / \partial G^j = 0$, we have

$$\frac{\partial \tilde{G}^j(\cdot)}{\partial x} = - \frac{\partial^2 W^j / \partial G^j \partial x |_{G^j = \tilde{G}^j(\cdot)}}{\partial^2 W^j / \left(\partial G^j\right)^2 |_{G^j = \tilde{G}^j(\cdot)}} \tag{A10}$$

for any $x \in \{\mu^j, \mu^k, G^k\}$. With $\partial^2 W^j/\partial G^j \partial \mu^j|_{G^j=\tilde{G}^j(\cdot)} < 0$, $\partial^2 W^j/\partial G^j \partial \mu^k|_{G^j=\tilde{G}^j(\cdot)} > 0$, and $\partial^2 W^j/\partial G^j \partial G^k|_{G^j=\tilde{G}^k(\cdot)} > 0$ if $\mu^k \in (0, q]$, the partial derivatives of $\tilde{G}^j(\cdot)$ follow. Part (iv) of the lemma follows from (A9) and $\partial W^j/\partial G^j|_{G^j=\tilde{G}^j(\cdot)} = 0$. *QED.*

Proof of Proposition 2

Without loss of generality, let us focus on $\mu^H \geq 0$, $\mu^F = 0$ in this proof. (The arguments for $\mu^H = 0$, $\mu^F \geq 0$ can be derived in an analogous way.) Moreover, let us restrict our analysis to a parameter domain that guarantees $G^H, G^F > 0$ in a Nash equilibrium with migration (under rational policy setting) as well as under bilateral policy coordination.

Denote by $\Delta W^j \equiv W^j|_{\mu^H=q} - W^j|_{\mu^H=0}$ the migration gains/losses of the median voter in country $j = H, F$ if at given G^H, G^F an equilibrium with migration instead of one without migration is realized. Moreover, let $\Delta W^c \equiv \Delta W^H + \Delta W^F$. According to (11), we obtain

$$\Delta W^H = -\frac{qG^H}{1-q}, \tag{A11}$$

$$\Delta W^F = \frac{qG^H}{1 + qG^H/G^F}. \tag{A12}$$

As a consequence, $\Delta W^c < 0$ for any positive G^H, so that governments prefer $\mu^H = \mu^F = 0$ to $\mu^H = q$ and $\mu^F = 0$ under coordination. Hence, for a given (G^H, G^F), W^c is highest if $\mu^H = \mu^F = 0$. Furthermore, partially differentiating (12) with respect to G^j gives

$$\frac{\partial W^c}{\partial G^j} = \frac{\partial W^H}{\partial G^j} + \frac{\partial W^F}{\partial G^j}. \tag{A13}$$

For $\mu^H = \mu^F = 0$, (A13) implies $\partial W^c/\partial G^j = \partial W^j/\partial G^j$, according to (11). Together with $\Delta W^c < 0$, this proves part (i) of Proposition 2.

Concerning part (ii) of the proposition, we know from the analysis in Sections 4 and 5 that for sufficiently small migration costs θ, education policies G_0^H, G_0^F are inconsistent with non-migration if migration decisions are based on go-abroad beliefs. Furthermore, the numerical results in Egger, Falkinger, and Grossmann (2007) show that in this case median voters may benefit from coordination of policies that allow for migration of high-skilled workers.

Finally, according to our analysis in Section 5, $G^H = \tilde{G}^H\left(q, 0, G^F\right)$, $G^F = \tilde{G}^F\left(0, q, G^H\right)$ in a non-cooperative policy equilibrium with $\mu^H = q$, $\mu^F = 0$. In view of (11) and (A13), we have

$$\frac{\partial W^c}{\partial G^H} = \frac{\partial W^H}{\partial G^H} + \frac{\partial W^F}{\partial G^H}, \tag{A14}$$

$$\frac{\partial W^c}{\partial G^F} = \frac{\partial W^F}{\partial G^F}. \tag{A15}$$

By definition, $\partial W^H/\partial G^H = \partial W^F/\partial G^F = 0$ at $G^H = \tilde{G}^H(q, 0, G^F), G^F = \tilde{G}^F(0, q, G^H)$. Moreover, differentiating (A9) and evaluating the resulting expression at $\mu^H = q$, $\mu^F = 0$ gives

$$\frac{\partial W^F}{\partial G^H} = \frac{q}{\left[1 + qG^H/G^F\right]^2} > 0. \tag{A16}$$

As a consequence, $\partial W^c\left(\cdot\right)/\partial G^H > 0$ at education policies $G^H = \tilde{G}^H\left(q, 0, G^F\right)$, $G^F = \tilde{G}^F\left(0, q, G^H\right)$, which implies that coordination gains exist. The numerical results in Egger, Falkinger, and Grossmann (2007) show that the direction of brain drain may be reversed. This completes the proof of part (iii). *QED.*

Appendix B. Derivation Details

Derivation of (10) and Properties of ρ_1^H

Substitute (7) for $\chi^H(q)$ into $\chi^H(q) = 1 + \theta$ and rewrite the equation in the form

$$\left(G^H/G^F\right)^{1-\beta}\left\{1 + \eta\phi^\beta\left(G^F\right)^{1-\beta}/\left(bA^F\right)\right\} = (1+\theta)A^H/A^F, \tag{A17}$$

where $\eta \equiv \frac{1+\theta}{1-q} - \frac{1}{1+qG^H/G^F} = \left(\theta + q\frac{1+G^H/G^F}{1+qG^H/G^F}\right)\frac{1}{1-q} > \theta$. Condition (A17) defines G^H/G^F as a decreasing function of G^F, starting at $G^H/G^F = \left[(1+\theta)A^H/A^F\right]^{1/(1-\beta)}$ for $G^F = 0$. With

$$\rho_1^H \equiv \left\{(1+\theta)/\left[1 + \eta\phi^\beta\left(G^F\right)^{1-\beta}/\left(bA^F\right)\right]\right\}^{1/(1-\beta)}, \tag{A18}$$

(A17) can be written as $G^H/G^F = \rho_1^H \left(A^H/A^F\right)^{1/(1-\beta)}$. Since $\eta > \theta$, $\rho_1^H < \rho_0^H$.

For the effect of a change in θ, set $B \equiv \phi^\beta \left(G^F\right)^{1-\beta} / \left(bA^F\right)$ and note that $\partial\rho_1^H/\partial\theta > 0$ if $1+\eta B > (1+\theta)\frac{B}{1-q}$. The latter condition is equivalent to $1 > \frac{B}{1+qG^H/G^F}$ which, according to (A6) and (A8), is further equivalent to the condition $(1-\tau^F)w_S^F > 0$.

The position and shape of $I_{H\rightarrow F}$ result from $\rho_1^H < \rho_0^H$ and the following facts. First, for $\theta > 0$, $\rho_1^H > 1$ at $G^F = 0$, according to (A18), implying that $I_{H\rightarrow F}$ lies above the EA line for low G^F. Second, ρ_1^H is decreasing in G^F, which explains the concave shape of $I_{H\rightarrow F}$ as shown in Figure 1.

Derivation of (13)

Recall that the utility of non-migrants is given by C^j, whereas the utility of migrants is $C^j/(1+\theta)$. From (3)-(5), $SW = \left(1-\mu^H\right) W^H + \mu^H (1-\bar{e}) \left(1-\tau^F\right) w_S^F G^H / (1+\theta) + \left(1-\mu^F\right) W^F + (1-\bar{e}) \left(1-\tau^H\right) w_S^H G^F / (1+\theta)$, where definition $W^j = (1-\tau^j)\, w_L^j$ has been used. Expression (13) follows from the definition of χ^H in (7) and the analogous one of χ^F, as well as the fact that $W^j = (1-\bar{e})\,(1-\tau^j)\, w_S^j G^j$, $j = G, F$, according to (4).

Proof of Proposition 3

The numerical results summarized in Tables 1 and 2 illustrate the differences between rational non-cooperative policies, bilateral coordination, and the social planner solution for the two different belief scenarios: stay-home beliefs in Table 1 and go-abroad beliefs in Table 2.[1] In the numerical exercise, we allow for corner solutions, i.e. zero education expenditures in one of the two economies. Proposition 3 follows immediately from the numerical results.

[1] As in the main text, $H \rightarrow F$ denotes that migration goes from H to F (vice versa for $F \rightarrow H$).

A^H/A^F		Non-cooperative	Coordination	Social Planner
1	G^H, G^F	104.17, 104.17	104.17, 104.17	104.17, 104.17
	$G^H + G^F$	**208.33**	**208.33**	**208.33**
	W^H, W^F	104.17, 104.17		
	$W^H + W^F$	**208.33**	**208.33**	
	SW	**208.33**	**208.33**	**208.33**
	migration	non-migration	non-migration	non-migration
3	G^H, G^F		937.50, 104.17	937.50, 104.17
	$G^H + G^F$		**1'041.67**	**1'041.67**
	W^H, W^F	no equilibrium		
	$W^H + W^F$		**1'041.67**	
	SW		**1'041.67**	**1'041.67**
	migration		non-migration	non-migration
8	G^H, G^F		6'666.70, 104.17	6'774.54, 0
	$G^H + G^F$		**6'770.87**	**6'774.54**
	W^H, W^F	no equilibrium		
	$W^H + W^F$		**6'770.87**	
	SW		**6'770.87**	**6'774.54**
	migration		non-migration	$H \to F$

Table 1: Comparison of non-cooperative policies, bilateral coordination, and the social planner solution if migration behavior is based on stay-home beliefs. ($\beta = 1/2, A^F = 50, \bar{e} = 1/3, q = 0.0015$ and $\theta = 0.09$)

A^H/A^F		Non-cooperative	Coordination	Social Planner
9	G^H, G^F		8'426.73, 114.20	8'560.32, 0
	$G^H + G^F$		**8'540.93**	**8'560.32**
	W^H, W^F	no equilibrium		
	$W^H + W^F$		**8'541.48**	
	SW		**8'541.49**	**8'560.32**
	migration		$F \to H$	$H \to F$
11	G^H, G^F	12'566.45, 108.86	12'604.09, 118.98	12'747.28, 0
	$G^H + G^F$	**12'675.30**	**12'723.07**	**12'747.28**
	W^H, W^F	12'585.32, 120.18		
	$W^H + W^F$	**12'705.51**	**12'707.90**	
	SW	**12'707.23**	**12'707.93**	**12'747.28**
	migration	$H \to F$	$F \to H$	$H \to F$
13	G^H, G^F	17'551.48, 0	17'604.11, 124.02	17'765.07, 0
	$G^H + G^F$	**17'551.48**	**17'728.13**	**17'765.07**
	W^H, W^F	17'577.85, 128.28		
	$W^H + W^F$	**17'706.12**	**17'707.56**	
	SW	**17'764.43**	**17'707.59**	**17'765.07**
	migration	$H \to F$	$F \to H$	$H \to F$

Table 2: Comparison of non-cooperative policies, bilateral coordination, and the social planner solution if migration behavior is based on go-abroad beliefs. ($\beta = 1/2, A^F = 50, \bar{e} = 1/3, q = 0.0015$ and $\theta = 0.01$)